The variability of PRICES as we discussed

But are the laws correct? For Aristotle, their power lay in their intuitive truth, their accord with our common sense. Yet Aristotle himself does not view statements as either true or false. In the *Metaphysics*, a few pages before announcing the Law of the Excluded Middle, he says, "The more and the less are still present in the nature of things" and adds that one who thinks 4 equals 5 is more correct than the one who thinks 4 equals 1,000.[13] In *De Interpretatione*, he uses the terms truer and falser. Even for him, truth has degrees.

In fact, Aristotle provides a wealth of arguments against his own axioms. The most famous involves ships clashing at sea. In Chapter 9 of *De Interpretatione*, he ponders the statement:

> There will be a sea battle tomorrow.

True? Not true?

A determinist would argue that, at this instant, it must be either true or false. Events are preordained, by divine hand or physical law. If we knew enough about celestial will or the sequence of causes, we could tell right now whether it were true or false. We don't, but it doesn't matter. At this moment, the statement is either true or false.

A nondeterminist would demur. Events are not preordained, so at this moment it may be impossible for us to know the answer, even with total knowledge. God may reconsider or random acts may jostle causality. The answer may simply not exist yet. If so, the statement is currently neither true not false, but inderterminate.

De Interpretatione is a dense, muddled work, and the crucial Chapter 9 is so opaque that scholars still contest its meaning. However, it appears to set forth these arguments, half-heartedly reject determinism, and conclude that "There will be a sea battle tomorrow" has an intermediate truth-value. A gap opens at the hairline. The Law of the Excluded Middle ebbs away.

This question exerted great historical influence. In the Middle Ages, both Christian and Muslim thinkers tended to accept only two values for the sea battle, yet some scholars took up where Aristotle left off. Duns Scotus (c. 1265-c. 1308) toyed with a third value straight from Aristotle, as did William of Ockham (c. 1300-1349), he of the famous razor. In the 20th century, this chapter would spur Jan Lukasiewicz to revolt against both laws.

The sea battle caused much spume to fly, but Aristotle made other important reservations too.

First, silhouette definitions are crucial to Aristotle's logic. Definitions are the hairline. They must cleanly separate items into *chair* and *not-chair*. Yet at the outset of *De Anima*, he admits the difficulty of finding any method of definition. It is a remarkable concession, for if we can't specify the hairline, classes blur at the edges and the whole two-values system becomes an artifice. He glozes over this problem in his works on logic, most likely for pragmatic reasons.

In the *Nicomachean Ethics,* Aristotle simply jettisons the axioms when discussing his famous golden mean. Here, an array of qualities possess a middle, usually the apex

of virtue. Take courage. It is a continuum. The craven, who fear too much, lie at one end and the foolhardy, who fear too little, at the other. A sensible person seeks the ground in between: valor.

But what are *cravenness, valor,* and *foolhardiness?* They resist iron encirclement, he admits. Yet he is not now in the realm of logic, but elsewhere, he says, and mist speak in terms which thwart sharp definitions. In fact, they aren't even necessary. "Our statement of the case will be adequate," he declares, "if it be made with all such clearness as the subject-matter admits; for it would be as wrong to expect the same degree of accuracy in all reasoning's as in all manufacturers." With topics such as character and ethics, "we must be content to indicate the truth roughly and in outline."[14] There is no hairline. Concepts wash over into each other, *valor* subtly merges into *foolhardiness* (i.e., *nonvalor*), and both laws dissolve.

He goes even further. Excess specificity, he says, leads to error, for "an educated person will expect accuracy in each subject only so far as the nature of the topic allows." [15] In other words, some subjects are simply vaguer than others, and we must fit the level of precision to the topic at hand. This statement comes very close to Zadeh's law of Compatibility.

Apples in the Refrigerator

Of course, on a day-to-day level, everyone constantly deals with propositions that are neither true nor false. Will it rain tomorrow? Regardless of determinism, the answer is uncertain for all practical purposes. Yet most people want to hear it, since the chance of rain affects their plans. In the latter 1600x, scholars began mathematicizing such uncertainties, and here too they followed Aristotle's laws. They created probability.

Probability is Aristotelian because it works with atoms of yes and no. A flipped coin is either heads or tails. A playing card falls into one of 52 categories and its identity is never vague. Thus, probability obeys the twin laws. A flipped coin can't be heads and tails, and it must be either heads or tails. A card can't be both the king of diamonds and the king, and must be one of them.

In mapping our vagueness, Max Black employed probability, perhaps because it was the dominant uncertainty tool at hand. He stated that if 80 percent of people say Tom is *tall*, then he is tall to 0.8 extent. At bottom lies the yes/no atom: Is tom *tall?*

This approach differs fundamentally from fuzziness and yields different results. Suppose Sally considers Tom 0.8 tall. She will probably always call him *tall*. Suppose a thousand people believe Tom is 0.8 tall. Ask them, "How likely are you to call Tom *tall?*" and every one might say, "100 percent." Yet they would all think he is 0.8 tall. Each is rounding off from 0.8 to 1, so the final figures describe word usage, not degree of truth.

Zadeh distinguished fuzziness from probability in his 1965 paper. Both describe uncertainty numerically, he said. However, probability treats yes/no occurrences, requires ignorance, and is inherently statistical. Fuzziness deals with degrees, does not require ignorance, and, he added, is completely nonstatistical.

For instance, to take Kosko's example, if we ask, "Is there an apple in the refrigerator?" we are dealing with probability. The answer might be 0.5, as in a coin toss. But suppose we know there is a half-eaten apple in the refrigerator?" That is a fuzzy question. The answer, however, is still 0.5.

Unlike fuzziness, Kosko notes, "Probability dissipates with increasing information." The more we know about the inside of the refrigerator, the less uncertain it is. If we open the door and gaze within, it becomes certainty: yes or no. Probability vanishes. If simply requires ignorance. That is why so much of it involves the future. Fuzziness, however, can coexist with total information. We can know everything possible about the half-eaten apple in the refrigerator, and the fuzziness remains.

Fuzziness plainly differs from probability, but also plainly resembles it, since both deal with degrees-one of truth, the other of likelihood or expectation. Hence, given the momentum probability has built up over the centuries, there is now tumult and shouting over their relation. Probability is today the great rival of fuzzy logic, and its champions claim it surpasses fuzzy logic in any task one can devise.